

# Calculations of Antenna Temperature, Horizontal Path Attenuation, and Zenith Attenuation Due to Water Vapor in the Frequency Band 150–700 GHz

R. W. McMILLAN, J. J. GALLAGHER, SENIOR MEMBER, IEEE, AND A. M. COOK, JR., STUDENT MEMBER, IEEE

**Abstract**—The results of calculations of antenna temperature at zenith, both with and without the sun viewed as a source, are given. Horizontal path and total zenith attenuation are also calculated. Each of these calculations was made over the frequency band 150–700 GHz, using data from the 24 water-absorption lines between 150 and 1000 GHz.

## I. INTRODUCTION

MOLECULAR RESONANCES of atmospheric water vapor have been studied for many years because this gas makes the major contribution to attenuation of microwaves and infrared radiation by the atmosphere. Many of these lines are very strong, with horizontal-path attenuations that exceed 10<sup>4</sup> dB/km, and exhibit strong pressure-broadening effects, so that attenuation is appreciable at frequencies several gigahertz removed from the center frequencies of the lines. The effects of these lines extend generally through the millimeter/submillimeter portion of the microwave spectrum, being most pronounced between about 2000 and 10  $\mu$ m. These absorptions make large portions of the submillimeter spectrum useless for communications or astronomical observations. However, the usefulness of these strong lines for remote sensing of the atmosphere from airborne, satellite, and ground-based stations is just beginning to be realized.

This paper presents the results of calculations of radiometric antenna temperatures at zenith, both with and without the sun viewed as a source. Calculations of horizontal path and total zenith attenuation are also given. The range of frequencies covered by these calculations is 150–700 GHz. Since the skirts of the water-vapor absorption lines extend to frequencies far removed from their center frequencies, all 24 lines in the frequency range 150–1000 GHz were considered in the calculations. The effects of oxygen and ozone were not included because the contributions of these molecules are considered to be negligible over the frequency range of interest for ground-based observations. Furthermore, absorptions due to the water dimer and the continuum were not treated because there exists no firm analytical basis for their inclusion in a calculation of this sort. For these reasons, more detailed calculations are required for specific system applications

over limited wavelength ranges. The calculations of horizontal attenuation show good agreement with the results of Zhevakin and Naumov [1].

## II. ABSORPTION CALCULATIONS

Van Vleck and Weisskopf [2] have shown that the attenuation coefficient  $\alpha$ , at frequency  $\nu$ , for a collision-broadened absorption line centered at frequency  $\nu_0$ , with linewidth parameter  $\Delta\nu$ , is given by

$$\alpha = \frac{8\pi^2 N n |\mu|^2 \nu^2 \exp(-E_i/kT)}{3c k T G} F(\nu) \quad (1)$$

where the other parameters are determined as discussed below. The parameter  $N$  is the number of molecules per unit volume and is

$$N = \frac{N_A \rho}{M} \quad (2)$$

in which  $N_A$  is Avogadro's number,  $\rho$  is the density of molecules, and  $M$  is the number of grams in a gram molecular weight. For water, this number is  $N = 3.346 \times 10^{16} \rho$ , where  $\rho$  is measured in grams/cubic meter. The factor  $|\mu|^2$  is the square of the dipole matrix element between transition states and is equal to  $\Sigma |\phi|^2 \mu_0^2$ , where  $\mu_0^2$  is the electric dipole moment. The factor  $\Sigma |\phi|^2$  is the line-strength parameter determined by King *et al.* [3], and  $\mu_0^2$  is  $3.39 \times 10^{-36}$  ESU from Van Vleck [4]. The statistical weighting factor  $n$  which accounts for nuclear spin is unity [5] for even rotational states and 3 for odd rotational states. In the exponential term,  $E_i$  is the energy of the lower transition state,  $k$  is Boltzmann's constant, and  $T$  is the absolute temperature at which the attenuation is measured. The partition function  $G$  has been calculated by Van Vleck [4] to be 170 at 293 K and varies with temperature as

$$G = KT^{3/2}. \quad (3)$$

Evaluation of the constant  $K$  from the previous values gives  $G = 0.0339T^{3/2}$ .

For these calculations, the line-shape factor  $F(\nu)$  was replaced by a factor calculated by Gross [6], which has been found to give better agreement with absorption-cell measurements

$$F(\nu) = \frac{4\nu\nu_0\Delta\nu}{(\nu_0^2 - \nu^2)^2 + 4\nu^2\Delta\nu^2} \quad (4)$$

in which the parameters are defined as before.

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The authors are with the Engineering Experiment Station, Georgia Institute of Technology, Atlanta, GA 30332.

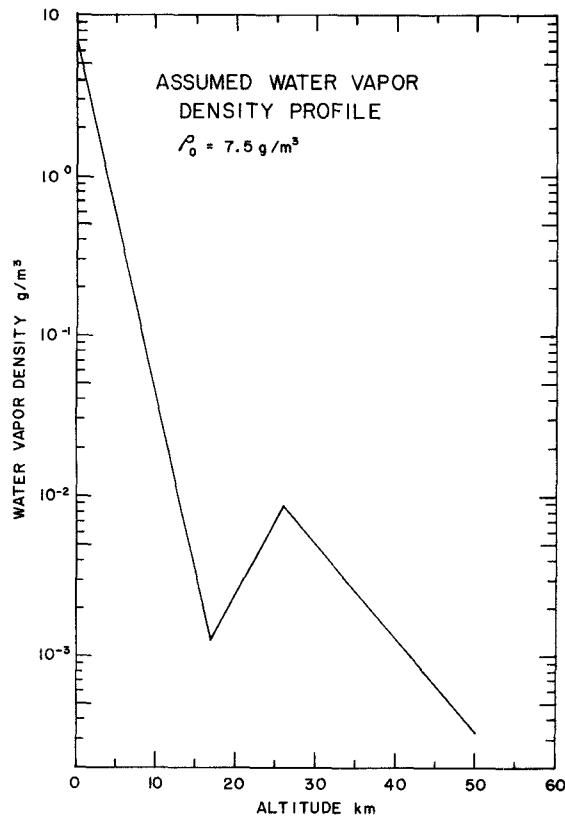


Fig. 1. Altitude dependence of water-vapor density for sea-level density of  $7.5 \text{ g/m}^3$ .

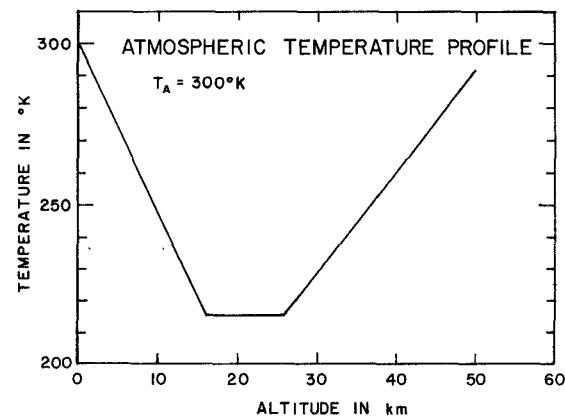


Fig. 2. Altitude dependence of air temperature for sea-level temperature of  $300 \text{ K}$ .

Since radiometric calculations must include the effects of altitude, the altitude dependence of terms in (1) must be considered. The density varies with altitude according to a relation given by Croom [7], [8], who based his altitude dependence on U.S. Weather Bureau data. This dependence is shown plotted in Fig. 1.

Barrett and Chung [9], based on U.S. Air Force data, give the altitude-dependent temperature profile shown in Fig. 2, and the pressure dependence as

$$P = 760 [10^{-3.05Z/50}]. \quad (5)$$

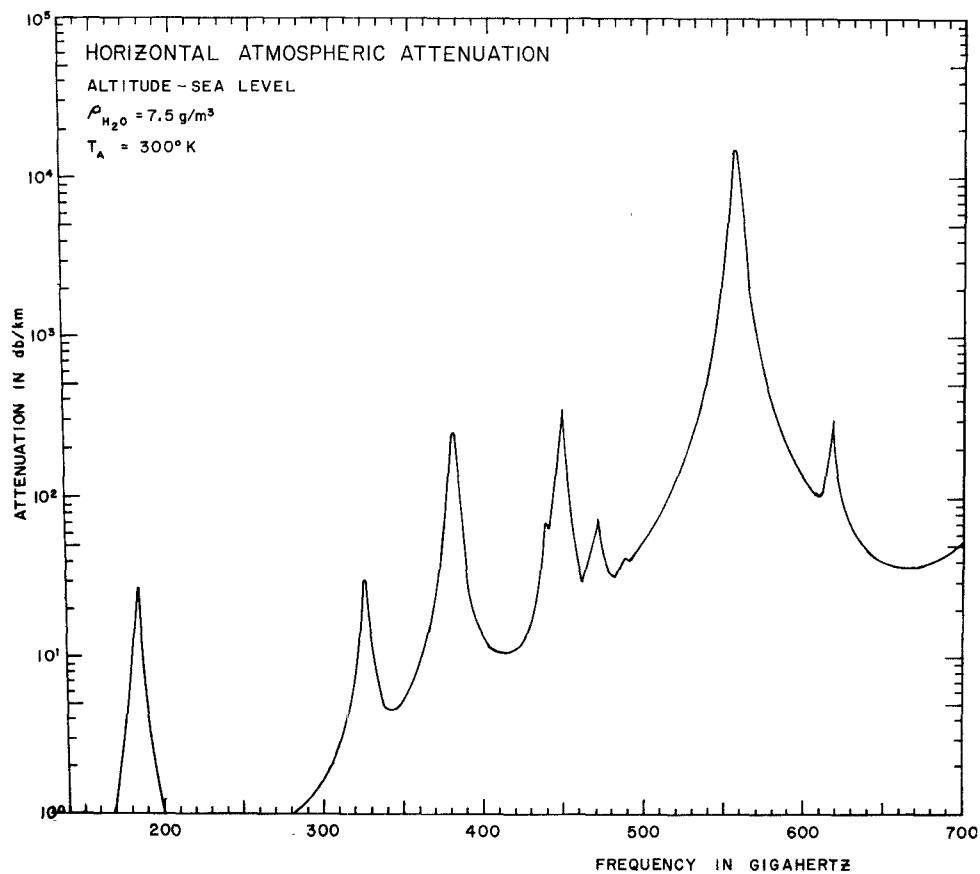


Fig. 3. Horizontal-path attenuation versus frequency at sea level.

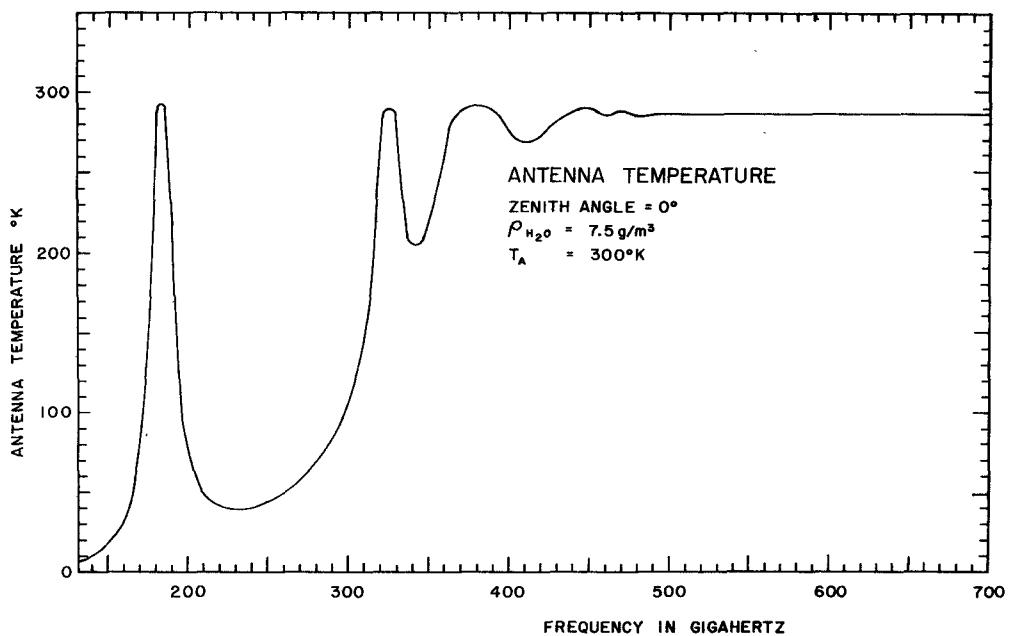


Fig. 4. Antenna temperature at zenith for sea-level altitude.

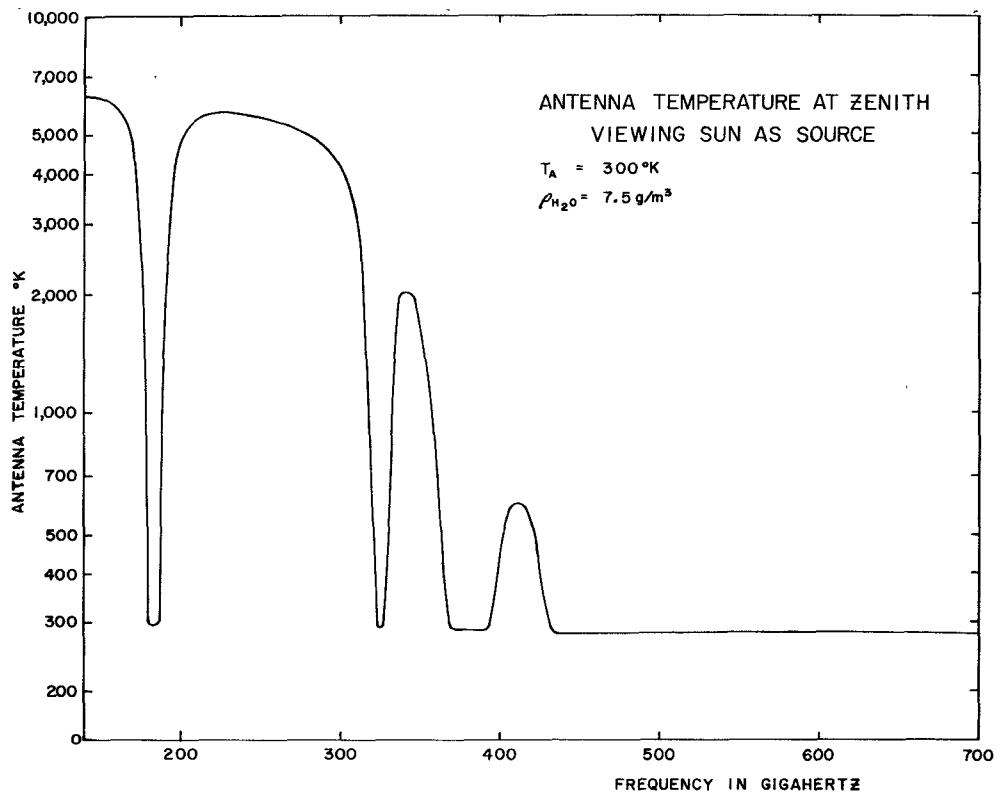


Fig. 5. Antenna temperature viewing the sun as a source at zenith from sea level.

Also from Barrett and Chung, the linewidth parameter is

$$\Delta\nu = \frac{\Delta\nu_0 \frac{P}{760} (1 + 0.0046\rho)}{\left(\frac{T}{318}\right)^{0.625}} \quad (6)$$

where  $\Delta\nu_0$  is the linewidth parameter at  $P = 760$  mm and  $T = 318$  K for small  $\rho$ .

Using these parameters with the indicated altitude, density, and temperature dependence, the attenuation in decibels/kilometer due to contributions from all of the water absorption lines below 1000 GHz was calculated in

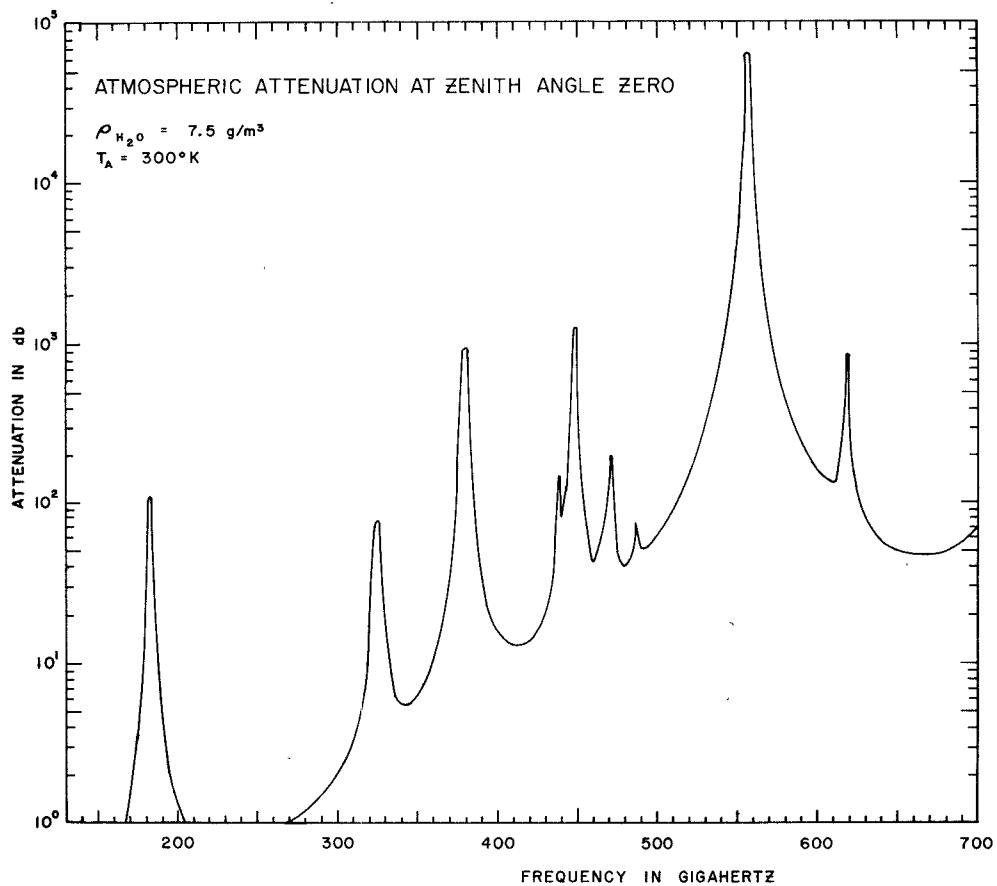


Fig. 6. Total zenith attenuation from sea level.

the frequency range from 150 to 700 GHz. The result of this calculation at sea level is shown in Fig. 3.

### III. SKY-TEMPERATURE CALCULATIONS

The background sky temperature  $T_b$ , measured by a radiometer at altitude  $h$  with an infinitesimally narrow beamwidth looking upward at zenith angle  $\theta$ , is [6]-[8]

$$T_b = T_s \exp \left( - \int_h^\infty \alpha(Z) \sec \theta \, dZ \right) + \int_h^\infty \alpha(Z) T(Z) \cdot \exp \left( - \int_h^Z (Z') \sec \theta \, dZ' \right) \sec \theta \, d\theta \quad (7)$$

in which  $T_s$  is the sun temperature, and  $T(Z)$  is the temperature of a stratum of atmosphere of thickness  $dZ$  located at altitude  $Z$ . In this equation, the first term is seen to be the brightness of the sun as viewed through the atmosphere, and the second term is the emission of the atmosphere. The sum of these two terms is the temperature that would be measured if the sun were viewed as a source through the atmosphere. Using the value of  $\alpha$  given by (1), this equation was integrated numerically up to an altitude of 50 km. In order to get good accuracy and minimize computer time, the stratum width was varied as follows:

$Z < 1.0 \text{ km}$	$\Delta Z = 2.5 \text{ m}$
$1.0 \text{ km} < Z < 4.0 \text{ km}$	$\Delta Z = 5.0 \text{ m}$
$4.0 \text{ km} < Z < 8.0 \text{ km}$	$\Delta Z = 10.0 \text{ m}$
$8.0 \text{ km} < Z < 16.0 \text{ km}$	$\Delta Z = 20.0 \text{ m}$
$16.0 \text{ km} < Z < 50.0 \text{ km}$	$\Delta Z = 50.0 \text{ m}$

The computer was programmed to read out emission temperature of the atmosphere, which is the temperature that would be measured by a radiometer pointing away from the sun; sky temperature, measured when the sun is viewed as a source; and zenith attenuation. The contributions of all of the water absorption lines between 150 and 1000 GHz to these temperatures were included in the calculations. The result of calculating emission temperature with the antenna pointing to the zenith is shown in Fig. 4. Emission temperature calculated with the sun viewed as a source is shown in Fig. 5, and total zenith attenuation is shown in Fig. 6.

### IV. CONCLUSIONS

Calculations of attenuation and emission temperatures due to water vapor in the atmosphere have been made between 150 and 700 GHz. Attenuation curves show windows at 230 and 400 GHz, and strong absorption elsewhere within this frequency band. Antenna temperature curves show no structure above 450 GHz, but strong structure due to the 183- and 325-GHz lines is evident at the lower frequencies. This result shows that the atmosphere appears to be a black body at near-ambient temperature at the peaks of the strong absorption lines and at frequencies greater than 450 GHz.

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# The Measurement of the Surface Resistivity of Evaporated Gold at 890 GHz

R. J. BATT, G. D. JONES, AND D. J. HARRIS

**Abstract**—A modified pyroelectric detector is used to measure the surface resistivity of evaporated gold at 890 GHz. The value of 0.65  $\Omega$  square yields a ratio of measured-to-theoretical surface resistivity of approximately 2.2.

## I. INTRODUCTION

THE ATTENUATION of a metal waveguide or the reflectivity of a mirror is determined by the surface resistivity of the wall of the guide or the mirror surface. To calculate the attenuation or reflectivity, the effective value of the surface resistivity is required at the operating frequency.

The attenuation for the fundamental  $\text{TE}_{10}$  transmission mode of a rectangular waveguide is given by [1]

$$\alpha(f)N/m = \frac{R_s}{120 b\pi} \left[ 1 - \left( \frac{f_c}{f} \right)^2 \right]^{1/2} \left[ 1 + \frac{2b}{a} \left( \frac{f_c}{f} \right)^2 \right] \quad (1)$$

where  $R_s$  is the surface resistivity. The theoretical value of  $R_s$  is related to the bulk resistivity  $\rho$  by the expression

$$R_s = (\pi f \mu_r \mu_0 \rho)^{1/2} \quad (2)$$

all the symbols having their usual meaning. The theoretical waveguide loss predictions using surface resistivity figures which are derived from dc values of bulk resistivity using

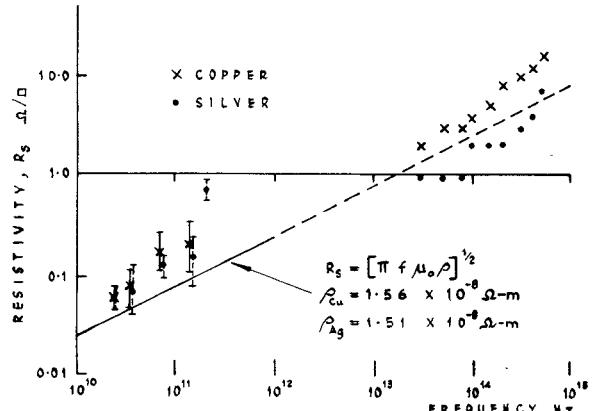


Fig. 1. Variation of surface resistivity of copper and silver with frequency.

(2) seriously underestimate the actual waveguide losses for frequencies greater than about 10 GHz. This apparent anomaly in surface resistivity has been investigated by many workers [2], [3] for various waveguide materials such as copper, silver, and brass in the microwave region up to frequencies of about 200 GHz. The reported ratios of measured-to-theoretical loss of up to 2.5 imply similar variations of effective-to-theoretical surface resistivity.

Fig. 1 summarizes some surface resistivity values for copper and silver deduced from the measured waveguide attenuations [2], [3] in the region 10–200 GHz. The theoretical variation with frequency is shown (solid line) assuming the bulk resistivities of copper and silver which are equal to within 3.5 percent. ( $\rho_{Cu} = 1.56 \times 10^{-8} \Omega \cdot \text{m}$ ,  $\rho_{Ag} = 1.51 \times 10^{-8} \Omega \cdot \text{m}$ .)

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R. J. Batt and G. D. Jones are with the Department of Electrical and Electronic Engineering, Portsmouth Polytechnic, Portsmouth, Hants, England.

D. J. Harris is with the Department of Physics, Electronics and Electrical Engineering, University of Wales Institute of Science and Technology, Cardiff, South Wales.